LOYOLA COLLEGE (AUTONOMOUS), CHENNAI – 600 034

M.Sc. DEGREE EXAMINATION – STATISTICS

FIRST SEMESTER – NOVEMBER 2009

ST 1813 - ANALYSIS

Date & Time: 04/11/2009 / 1:00 - 4:00 Dept. No. Max. : 100 Marks

SECTION – A

Answer ALL the following questions (10 x 2 = 20 marks)

1. Define a convergent sequence in a metric space.
2. State any two conditions to be satisfied by a norm.
3. Define an isometry
4. Define a compact metric space.
5. Explain the symbols O and o.
6. State any one property of a uniformly convergent sequence of functions which is also assured for the limiting function.
7. State the limit form of comparison test.
8. When is a function said to be Riemann-Stieltjes integrable with respect to a monotonic increasing function?
9. Define linear derivative of a function f: $\mathbb{R}^m \to \mathbb{R}^n$.
10. State a necessary and sufficient condition for a complex function to be differentiable at a point.

SECTION – B

Answer any FIVE questions (5 x 8 = 40 marks)

11. Show that two metrics ρ and σ on the same set **X** are equivalent if there exist constants $\lambda > 0$ and $\mu > 0$, such that $\lambda\, \rho(x, y) \le \sigma(x, y) \le \mu\, \rho(x, y)\ \forall\, x, y \in \mathbf{X}$. Give an example to show that the converse is not true.

12. (a) State and prove Cauchy's inequality concerning inner products.
    (b) Show that $\| x + y \|^2 + \|x - y \|^2 = 2 (\| x \|^2 + \| y \|^2)$

13. Show that a necessary and sufficient condition for a function f : (**X**, ρ) → (Y, σ) to be continuous is that, whenever **G** is an open set in **Y**, $f^{-1}(G)$ is open in **X**.

14. Test for absolute convergence of the following series:
    (a) $\sum_{n=1}^{\infty} \frac{n!}{(1-i\,n)^n}$ (b) $\sum_{n=1}^{\infty} \frac{n!(3n)!}{(4n)!} 10^n$ (4 + 4)

15. Show that for a power series $\Sigma\, a_n z^n$, the radius of convergence is $R = 1 / \limsup | a_n |^{1/n}$.
    Hence, find the radius of convergence of the power series $\sum_{n=2}^{\infty} \frac{z^n}{n\,(\log\, n)^2}$

16. State and prove a necessary and sufficient condition for a function f to be Riemann-Stieltjes integrable with respect to a monotonic increasing function.

17. Prove that Riemann-Stieltjes integral is additive with respect to the integrand.

18. Establish the relation between linear derivative and the matrix of partial derivatives of a function from $\mathbb{R}^m \to \mathbb{R}^n$.

SECTION – C

Answer any TWO questions (2 x 20 = 40 marks)

19. (a) State the three definitions of limit point of a set and establish their equivalence.
(b) Find the set of all interior points of the set E = {0, 1, 2, ...} in ($\mathbb{R}$, ρ) and in ($\mathbb{R}$, *d*) where ρ is the usual metric and *d* is the discrete metric.

(15 + 5)

20. (a) Let $\langle u_n \rangle$ be any sequence of real numbers and $\langle v_n \rangle$ be a sequence of positive real numbers. Let $s_n = u_1 + u_2 + \cdots + u_n$ and $t_n = v_1 + v_2 + \cdots + v_n$ such that $t_n \to \infty$. Prove that $\underline{\lim}\ \frac{u_n}{v_n} \le \underline{\lim}\ \frac{s_n}{t_n} \le \overline{\lim}\ \frac{s_n}{t_n} \le \overline{\lim}\ \frac{u_n}{v_n}$
(b) Using (a) and any other result (to be quoted) , deduce that if $\langle x_n \rangle$ is a sequence of positive numbers, $\underline{\lim}\ x_n \le \underline{\lim}\ (x_1 \cdot x_2 \cdots x_n)^{1/n} \le \overline{\lim}\ (x_1 \cdot x_2 \cdots x_n)^{1/n} \le \overline{\lim}\ x_n$

(12 + 8)

21. Establish Weierstrass Approximation Theorem (by proving all the required lemmas).

22. (a) Establish the equation for 'integration by parts'.
(b) An ellipse in $\mathbb{R}^3$ is given by the equations

$$g_1 (x, y, z) = 2 x^2 + y^2 - 4 = 0$$
$$g_2 (x, y ,z) = x + y + z = 0$$

Find the points on the ellipse which are nearest to and furthest from the y –axis.

(8+12)

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